Announcements

Prelim 1: Thursday, Feb 12. Statler Auditorium. 7:30-9pm

Greedy and Dynamic Programming: Covers hw1-2, sections weeks1-2, lectures through 2/4

Other prelim info and practice questions, hw and section materials and solutions posted on Canvas

Hw2 grades will be opened tonight & quiz also

Next plans: Feb break, then very short hw due Friday as usual

Plan today: quick review of hw question 3, and then start new material

Event planning

Input: $[s_i, f_i]$ price they offer to pay p_i

What subproblems to use:

smaller problems you want to solve

$Opt(i) = max(Opt(i-1), p_i + ?)$

not using i

using i

natural idea

1) Opt(i) = value of best solutions with items 1…i:

2) Opt(i) = value of best solutions with items 1…i with item i included:

Required times for switching

$r(1,3) = 2$ hours

$r(2,3) = 15$ mins

Option (1) not working

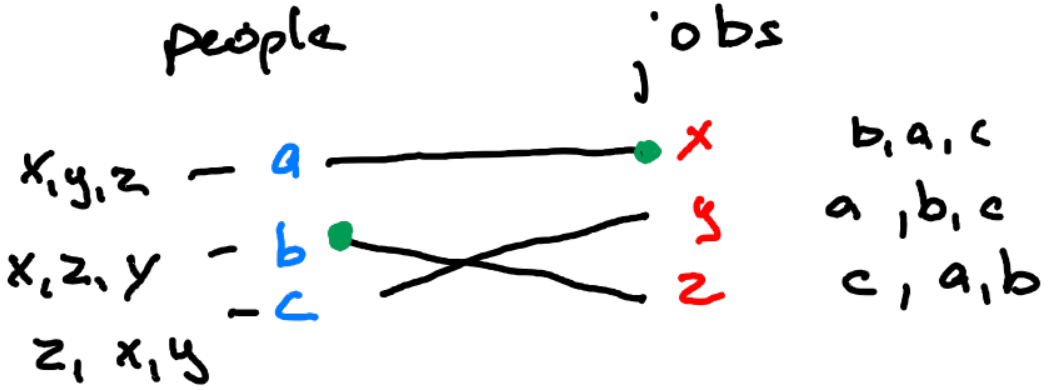
$Opt(1) = 20$

$Opt(2) = 20$

$Opt(3) = max(Opt(2), 100 +$ need item here that is not in our DP table)

Stable Matching I:

The problem and its applications



Is solution above OK?

(b, x) pair : prefer each other to their partner

call instability

may cause solution to be abandoned

used for medical resident matchings

NY city high school

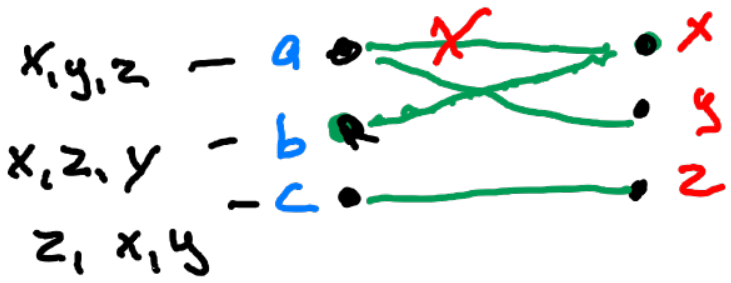
matching : assigning people to jobs
- at most one job / person
- at most one person / job

matching is perfect if all jobs / people are assigned

Question:
is there always a stable solution & can we find it

stable = no instability

The Gale Shapley Algorithm



b, a, c
a, b, c
c, a, b

steps:

a → x

b → x ⇒ a gets rejected

a → y

c → z

While there an applicant who is unmatched & there is a hospital they have not applied to

- apply to top option where they were not previously rejected
- keep best of those that applied, reject rest

end while

Return current matching

Properties of the Gale Shapley algorithm

(1) keep getting rejected & have to apply to less & less preferred jobs

* (2) hospitals: current match only improves once matched will remain matched

Assume n applicant & n jobs

Does this find a perfect matching

Claim: yes it does.

Proof: by contradiction: ∃ applicant a & job x both unmatched

a •

• x

Algorithm end: a applied to job x but hospital with any applicant ever remain matched a contradiction

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How many offers can be made overall running the Gale Shapley Algorithm with n applicants and m positions

A. $O(n)$

B. $O(n+m)$

C. $O(nm)$

D. $O(n^2m^2)$

E. Can be infinite

While there ∃ unmatched applicant
...
- apply to next job on their list
- hospitals keep their favorite *
  reject all others

end while

applicant only apply to every school once

Does Gale Shapley produce a Stable solution

Is it better to propose or to be proposed to?

Friday